

HIGH-MAGNETIC-FIELD CORRECTIONS TO CESIUM HYPERFINE STRUCTURE *

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Corrections to the Breit-Rabi formula for the ground state of ^{133}Cs are calculated. For a recently proposed high-magnetic-field frequency standard, the corrections amount to a few parts in 10^{12} .

De Marchi has proposed a cesium atomic-beam frequency standard based on the ($M_F = -1$, $\Delta M_F = 0$) transition, at the magnetic field ($B \approx 82$ mT) where the derivative of the frequency with respect to B is zero,¹ and has carried out preliminary experiments.² In order for this to be a *primary* frequency standard, it is necessary to relate the measured frequency to that of the zero-field transition frequency ν_0 , which defines the SI unit of time. The Breit-Rabi formula predicts the frequency to be $\sqrt{15/16}\nu_0 = 8\,900\,727\,438.257$ Hz.¹ For higher accuracy, additional terms must be taken into account.

Three corrections to the Breit-Rabi formula have been measured for the ground-state hyperfine structure of rubidium by Larson and coworkers.³⁻⁵ These are the dipole diamagnetic shift, the quadrupole diamagnetic shift, and the hyperfine-assisted Zeeman shift. These terms have not yet been measured for cesium.

The dipole diamagnetic shift results from a cross term between the contact hyperfine interaction and the atomic diamagnetic interaction in second-order perturbation theory. This leads to a shift, proportional to B^2 , in the dipole hyperfine constant A . I evaluated the perturbation sum over states, including the continuum, by solving an inhomogeneous differential equation for the perturbed wavefunction. I used an empirical potential for the valence electron derived by Klapisch.⁶ The result is $\delta A/A = 5.46 \times 10^{-10} B^2$, where B is expressed in teslas. This leads to an increase in the ($M_F = -1$) field-independent transition frequency of 33 mHz. Previous calculations have been reported by Bender⁷ and by Ray *et al.*⁸ In order to check the method, I used Klapisch's potential to calculate $\delta A/A$ for rubidium. The result is 10% below the experimental value. I estimate an accuracy of around 20% for the present calculation in cesium.

The quadrupole diamagnetic shift results from a cross term between the nuclear quadrupole hyperfine interaction and the electronic diamagnetic interaction in second-order perturbation theory. The interaction is diagonal in M_I and M_J . The perturbation sum over states is the same one that appears in the calculation of the quadrupole shielding factor γ_∞ .⁴ Using the value $\gamma_\infty(\text{Cs}^+) = -86.8$,⁹ I obtain the result

$$\langle M_I, M_J | H_{\text{QD}} | M_I, M_J \rangle = -4.6 \times 10^{-5} B^2 \frac{[3M_I^2 - I(I+1)]}{I(2I-1)}. \quad (1)$$

The result is in hertz if B is expressed in teslas. A similar calculation for rubidium yields a coefficient in good agreement with experiment.⁴ For cesium, the estimate of the coefficient

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for the shift is probably accurate within around 30%. However, for the special case of the ($M_F = -1$) field-independent transition, the shift vanishes. The two states involved in the transition are $(|-1/2, -1/2\rangle \pm |-3/2, 1/2\rangle)/\sqrt{2}$, in the $|M_I, M_J\rangle$ basis, where $+$ refers to the higher-energy state and $-$ to the lower-energy state. Hence, the states are shifted by the same amount, since they have the same M_I -content.

The hyperfine-assisted Zeeman shift was explained by Fortson as a third-order perturbation, in which the contact hyperfine interaction acts twice and the electronic Zeeman interaction acts once.¹⁰ Fortson gave diagonal matrix elements in the $|M_I, M_J\rangle$ basis. For the ($M_F = -1$) field-independent transition, it is necessary to extend Fortson's calculation to include off-diagonal matrix elements. In the $|(IJ)F, M_F\rangle$ basis, the nonzero matrix elements for ^{133}Cs are

$$\langle 3, M_F | H_{\text{HAZ}} | 4, M_F \rangle = 4h\beta B \sqrt{16 - M_F^2}. \quad (2)$$

Fortson used quantum-defect theory to make a semi-empirical estimate for β in rubidium; it agreed with experiment within 10%. The same method, applied to cesium, yields $\beta = 88$ mHz/T. For the ($M_F = -1$) field-independent transition, the shift at $B = 82$ mT is $-2\sqrt{15}\beta B = -56$ mHz. This estimate of the shift is probably accurate within around 30%.

To achieve a theoretical error of 10^{-14} in the total frequency, which is the projected accuracy of the high-field frequency standard,¹ the larger shifts must be known within less than 1%. This could possibly be done by *ab initio* calculations or by experiments carried out at much higher magnetic field, similar to those of Larson and coworkers in rubidium.

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